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CONVECTION HEAT TRANSFER COEFFICIENTS FOR TURBULENT FLOW BETWEEN PARALLEL PLATES WITH UNEQUAL HEAT FLUXES

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Abstract—This paper contains a semi-theoretical analysis of asymmetric heat transfer in fully developed two dimensional turbulent flow between parallel walls.

An extension of the analogy between the transfer of heat and the transfer of momentum (due to Mizushina), is used to determine the heat transfer coefficients for the respective boundaries. The results of the analysis are presented in the form of working formulae from which the relative magnitudes of the heat transfer coefficients may be determined. The formulae may be applied to practical engineering problems, but care is necessary because the assumptions made in the theory impose restrictions on the Prandtl and Reynolds number ranges for which the predictions are acceptable.

It is immediately obvious from the graphical presentation of the results for a particular fluid, that the heat flux ratio is an important additional parameter, and that cognizance of this should be taken in cases of non-uniform heating.

By its nature, the present analysis is preliminary, and an experimental programme has been devised to study the general problem of asymmetric heat transfer and to test the theory.

While channel flow has been considered, the results are thought to be valid for axisymmetric flow in annuli with small outside diameter/inside diameter ratios.

Résumé—Cet article donne une analyse semi-théorique de la transmission de chaleur dans un écoulement turbulent bidimensionnel dissymétrique, pleinement établi, entre deux murs parallèles.

Une extension de l'analogie entre transfert de chaleur et transfert de quantité de mouvement (due à Mizushina) est utilisée pour déterminer les coefficients de transmission de chaleur pour les deux parois. Les résultats de l'analyse sont présentés sous la forme d'une formule pratique à partir de laquelle on peut déterminer les valeurs relatives des coefficients de transmission de chaleur. La formule peut être appliquée à des problèmes techniques réels, mais il faut prendre garde au fait que les hypothèses faites dans la théorie imposent des restrictions dans les domaines de nombres de Reynolds et de Prandtl pour lesquels les résultats sont acceptables.

Il apparait immédiatement sur la présentation graphique des résultats pour un fluide particulier, que le rapport des flux de chaleur est un paramètre supplémentaire important, qu'il faudra connaître dans les cas de chauffage non-uniforme.

Par sa nature, l'étude actuelle est préliminaire, un programme expérimental a été établi pour étudier le problème général d'une transmission de chaleur assymétrique et vérifier la théorie.

Bien que l'écoulement dans un conduit ait été considéré ici, on pense que les résultats sont également valables pour un écoulement à symétrie axiale dans des anneaux, dont les rapports diamètre extérieur sur diamètre intérieur sont petits.

Zusammenfassung—Die Arbeit enthält eine halbtheoretische Untersuchung über den unsymmetrischen Wärmeübergang in vollentwickelter zweidimensionaler turbulenter Strömung zwischen parallelen Wänden.

Eine Erweiterung der Analogie zwischen Wärme- und Impulsübertragung (nach Mizushina) dient zur Ermittlung der Wärmeübergangskoeffizienten an den beiden Wänden, die in Form einer Gebrauchsgleichung für den relativen Wert des Koeffizienten mitgeteilt werden. Bei der Anwendung ist der begrenzte Bereich der Prandtl- und Reynoldszahl zu beachten.

Aus der graphischen Darstellung der Ergebnisse ergibt sich, dass das Verhältnis der Wärmestromdichten bei ungleichförmiger Beheizung von besonderer Bedeutung ist. Zur Prüfung dieser vorläufigen Theorie wird ein Versuchsprogramm angegeben. Die Theorie ist auch auf Ringkanäle mit kleinen Durchmesserverhältnissen anwendbar.

Аннотация—Эта статья содержит полутеоретический анализ асимметричного теплообмена в полностью развитом двухмерном турбулентном потоке между параллельными стенками. Для определения коэффициентов теплообмена используется (в соответствтующих границах) аналогия между теплопереносом и переносом количества движения (по Мизушина).

Результаты анализа представлены в виде расчётных формул, на основании которых можно определить относительные величины коэффициентов теплообмена. Формулы можно применить и к практическим ниженерным задачам, но при этом необходима осторожность, т.к. сделанные в теории предположения налагают ограничения на диапазоны чисел Прандтля и Рейнольдса, для которых эти допущения приемлемы.

Из графического представления результатов для одной жидкости сразу же становится очевидным, что соотношение тепловых потоков является важным дополнительным параметром и что знание этого фактора нужно учитывать в случаях неравномерного нагрева.

Настоящий анализ по своей природе является предварительным. Намечена программа экспериментальных работ для изучения общих проблем асимметричного теплопереноса и проверки теории.

Рассматривался поток по каналу. Предполагается, что эти результаты окажутся справедливыми и для осесиметричного потока в круглых каналах с малыми величинами отношения между наружным и внутренним диаметрами.

NOMENCLATURE

$d_e = 2s$ = equivalent diameter;
c = specific heat at constant pressure;
h = convection heat transfer coefficient;
ϵ = eddy diffusivity of heat;
$\alpha = K/c\rho$ = thermal diffusivity;
K = thermal conductivity;
ρ = fluid density;
s = distance between walls;
q = specific heat flux;
u, $\bar{u}$, u_τ = local velocity, average velocity, friction velocity;
y = wall distance;
μ = viscosity;
τ = shear stress;
T = temperature;
$u^+ = u/u_\tau$ = friction velocity parameter;
$y^+ = yu_\tau\rho/\mu$ = friction distance parameter;
x = distance in flow direction;
γ = a number;
$Re = d_e\rho\bar{u}/\mu$ = Reynolds number;
$Pr = c\mu/K$ = Prandtl number;
$Nu = hd_e/K$ = Nusselt number.

Suffixes

w = wall;
1 = outside boundary of sublayer;
b = bulk or mixed mean.

INTRODUCTION

THERE are a number of practical applications in which the thermal flux is non-uniform across the flow section. Fig. 1(a) shows two such arrangements which are of immediate interest. For example, thermal insulation may be provided by means of a single coolant flowing in forced convection in ventilation spaces formed by parallel walls or concentric shells adjacent to the heated element [1]. A ventilated space in conjunction with conventional insulating materials is an alternative scheme for reducing heat loss to the surroundings [2]. These arrangements do not necessarily reduce the heat loss from the heated element compared with conventional insulation, but that fraction of the heat which is lost to the coolant is both channelled and controlled. Another practical case involving asymmetric heat flow which might be envisaged is when various coolants are employed in separate flow channels. The coolants employed would depend, amongst other things, on problems of heat removal and heat insulation, and in such a scheme, a sandwiched fluid could provide an additional mechanical shield between two other fluids.

When a fluid (or fluids) which is transparent for thermal radiation is used, heat transfer due to radiation occurs between the hot and cold boundary walls. This depends on the temperature of the walls, the geometry of the flow passage, and the nature of the surfaces employed. In the following analysis thermal radiation is neglected, but its inclusion is simple should

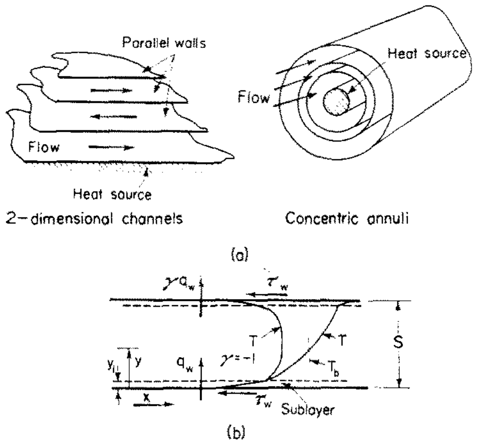

FIG. 1. Asymmetric heat transfer.

conditions make it significant. At sufficiently high coolant flow rates, free convection is swamped and this mechanism of heat transfer is also neglected.

THEORY

The system to be analysed is shown in Fig. 1(b). The fluid is in well developed turbulent flow between two parallel walls distance s apart. At one wall, thermal flux q_w is transferred to the fluid, while γq_w is transferred from the fluid at the opposite boundary. A diagrammatic temperature distribution for the system is shown for the case of $1 > \gamma > 0$; the shape of the profile is discussed in a later section. A heat balance indicates that $q_w(1 - \gamma)$ heat per unit surface area per unit time is channelled with the fluid.

The determination of the two convection heat transfer coefficients requires a knowledge of the temperature distribution in the y direction, and this in turn depends on the velocity distribution when an analogy between the transfer of heat and transfer of momentum is used. For channel flow, Knudsen and Katz [3] suggest that the equation

$$u^+ = 6{\cdot}2 \log_{10} y^+ + 3{\cdot}6$$

correlates the velocity distribution data in the turbulent region.

For the laminar sublayers adjacent the walls, the equation

$$u^+ = y^+$$

may be used, and from these equations the extent of the laminar sublayer, which presents the greater resistance to heat flow, may be determined by solving

$$y_1^+ = 6{\cdot}2 \log_{10} y_1^+ + 3{\cdot}6$$

when,

$$y_1^+ (= u_1^+) = 9{\cdot}74$$

Fig. 2 shows the simplified form of the universal velocity profile for channel flow. No account is taken of the buffer or transition zone.

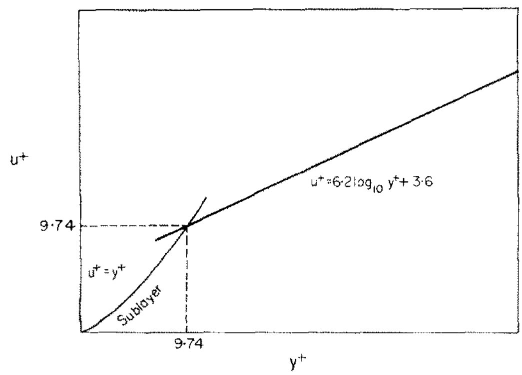

FIG. 2. Velocity distribution.

The convection heat transfer coefficient h which is to be determined, is defined by the equation

$$h = \frac{q_w}{T_w - T_b}$$

(For the other boundary, the appropriate heat flux γq_w and appropriate temperature difference would be used in the definition.)

For terminal conditions with heat fluxes q_w and γq_w uniform lengthwise, and constant heat transfer coefficients, the temperature differences between the fluid and the walls are constant, and furthermore

$$\frac{dT}{dx}\left(= \frac{dT_b}{dx} = \frac{dT_w}{dx}\right)$$

is constant and independent of position in the channel. The temperature differences between the walls and the fluid are determined by calculating the sublayer and turbulent core temperature drops separately.

Considering first the q_w wall.

In the sublayer, the transfers of heat and momentum are on a molecular scale, when

$$q_w = \frac{K(T_w - T_1)}{y_1}$$

$$\tau_w = \mu \frac{u_1}{y_1}$$

and with

$$u_1^+ = \frac{u_1}{u_\tau} = 9{\cdot}74$$

the temperature drop is

$$(T_w - T_1) = \frac{9{\cdot}74\, q_w Pr}{c\rho u_\tau} \tag{1}$$

Unlike fully developed pipe flow, the distributions of shear stress and heat flux in the turbulent core are not similar functions of the wall distance, in which case, the Reynolds analogy between the transfer of heat and the transfer of momentum does not apply. For the temperature difference $(T_1 - T_b)$, use can be made of Mizushina's [4] extension of the analogy which is outlined in a previous paper [5] dealing with a channel with one wall adiabatic, which is a particular case of asymmetric heat transfer. When $\gamma = -1$, the heating is symmetrical and the velocity and temperature profiles are similar in shape, so that the Reynolds analogy may be used giving the thermal resistance of the turbulent core

$$\frac{T_1 - T_b}{q_w} = \frac{\bar{u} - u_1}{c\tau_w} \tag{2}$$

This may also be obtained from the heat balance equations

$$-c\rho(\epsilon + \alpha)\frac{\partial T}{\partial y} = \bar{u}c\rho\frac{\partial T}{\partial x}\left(\frac{s}{2} - y\right)$$

and

$$q_w = \bar{u}c\rho\frac{\partial T}{\partial x}\left(\frac{s}{2} - y_1\right)$$

(for unit width of channel, and assuming that the velocity in the core is constant across the flow section and equal to the mean velocity $\bar{u}$). With the assumption that the eddy conductivity $(\epsilon + \alpha)$ is constant, and since $\partial T/\partial x$ is independent of y, these equations may be solved for the temperature, T in the turbulent core.

$$T = T_1 - \frac{q_w}{c\rho(\epsilon + \alpha)}\left\{\frac{sy}{2} - \frac{y^2}{2} - \frac{sy_1}{2} + \frac{y_1^2}{2}\right\}$$

and furthermore, with

$$T_b = \frac{1}{s/2 - y_1}\int_{y_1}^{s/2} T\mathrm{d}y$$

The thermal resistance of the turbulent core is

$$\frac{T_1 - T_b}{q_w} = \frac{2}{c\rho(\epsilon + \alpha)(s - 2y_1)^2} \left\{\frac{s^3}{12} - \frac{s^2 y_1}{2} + sy_1^2 - \tfrac{2}{3}y_1^3\right\} \tag{3}$$

When the laminar sublayer thickness y_1 is small compared with the channel width s, and since the Reynolds analogy is valid for $\epsilon \gg \alpha$ then equations (2) and (3) combined give

$$\left(\frac{T_1 - T_b}{q_w}\right) = \frac{\bar{u} - u_1}{c\tau_w} \doteqdot \frac{s}{6c\rho\epsilon}$$

Now considering the general case of asymmetric heating, with the same assumptions, the heat balance equations are

$$(q_w - \gamma q_w) = \bar{u}c\rho\frac{\partial T}{\partial x}(s - 2y_1)$$

and,

$$-c\rho(\epsilon + \alpha)\frac{\partial T}{\partial y} - \gamma q_w = \bar{u}c\rho\frac{\partial T}{\partial x}(s - y - y_1)$$

Solving, the temperature for this case is given by

$$T = T_1 - \frac{q_w(sy - y^2/2)}{c\rho(\epsilon + \alpha)(s - 2y_1)} - \frac{\gamma q_w y^2/2}{c\rho(\epsilon + \alpha)(s - 2y_1)} \tag{4}$$

Neglecting the terms in y_1 in the numerators, and with

$$T_b = \frac{1}{s - 2y_1}\int_{y_1}^{s - y_1} T\mathrm{d}y$$

the thermal resistance of the turbulent core for asymmetric heat transfer is

$$\frac{T_1 - T_b}{q_w} \doteqdot \frac{(2+\gamma)s}{6c\rho(\epsilon + \alpha)}$$

or

$$\frac{T_1 - T_b}{q_w} \doteqdot (2+\gamma)\frac{\bar{u} - u_1}{c\tau_w} \tag{5}$$

Hence the resistance of the turbulent core, referred to the q_w wall for asymmetric heating, is approximately $(2+\gamma)$ times that for symmetrical heating.

By adding equations (1) and (5), the total temperature difference between the q_w wall and the fluid is

$$T_w - T_b = \frac{9{\cdot}74\, q_w Pr}{c\rho u_\tau} + \frac{q_w(2+\gamma)(\bar{u} - u_1)}{c u_\tau^2 \rho}$$

Now the Blasius equation

$$\tau_w/\tfrac{1}{2}\rho\bar{u}^2 = 0{\cdot}079\, Re^{-1/4},$$

or

$$u_\tau^2 = 0{\cdot}0395\, Re^{-1/4}\, \bar{u}^2,$$

suitably correlates the wall shear and the Reynolds number of the flow, and since

$$Nu = \frac{Re\, Pr\, q_w}{c\rho\bar{u}(T_w - T_b)} \text{ by definition,}$$

$$Nu = \frac{0{\cdot}1986\, Re^{7/8}\, Pr}{5{\cdot}03(2+\gamma)\, Re^{1/8} + 9{\cdot}74\,[Pr - (2+\gamma)]} \tag{6}$$

At this point, it should be noted that with $\gamma = -1$, i.e. for symmetrical heating, equation (6) becomes similar in form to the heat transfer equation for pipe flow.

The convection heat transfer coefficient for the γq_w wall may be determined in an identical manner with the heat balance equations

$$(q_w - \gamma q_w) = \bar{u} c \rho \frac{\partial T}{\partial x}(s - 2y_1)$$

and,

$$c\rho(\epsilon + \alpha)\frac{\partial T}{\partial y} - \gamma q_w = \bar{u} c \rho \frac{\partial T}{\partial x}(y - y_1)$$

with y the γq_w wall distance.

The temperature T is given by

$$T = T_1 + \frac{q_w(1-\gamma)y^2/2}{c\rho(\epsilon+\alpha)(s - 2y_1)} + \frac{\gamma q_w y}{c\rho(\epsilon + \alpha)} \tag{7}$$

and the thermal resistance of the turbulent core referred to that wall becomes

$$\frac{T_b - T_1}{\gamma q_w} = (2 + 1/\gamma)\frac{s}{6c\rho\epsilon} \doteqdot (2 + 1/\gamma)\frac{\bar{u} - u_1}{c\tau_w}$$

where

$$Nu = \frac{Re\, Pr\, \gamma q_w}{c\rho u(T_b - T_w)} =$$

$$\frac{0{\cdot}1986\, Re^{7/8}\, Pr}{5{\cdot}03(2 + 1/\gamma)\, Re^{1/8} + 9{\cdot}74\,[Pr - (2 + 1/\gamma)]} \tag{8}$$

DISCUSSION

The temperature distribution for asymmetric heat transfer in a two-dimensional channel, has been determined by considering each wall in turn. Equations (4) and (7) for the q_w wall and γq_w respectively, are of the same form as they should be. A typical temperature profile is sketched in Fig. 1(b) showing d^2T/dy^2 to be positive at all points in the turbulent core. This is partly the result of assuming constant eddy diffusivity ϵ [and hence conductive $(\epsilon + \alpha)$] in that region. (The eddy diffusivity ϵ is a function of wall distance and Reynolds number.) In reality, for positive values of γ, a point of contraflexure is to be expected near the centre of the channel. For $\gamma = 1$, the temperature T is linear in y in the core, and the whole thermal flux is transferred from one wall to the other, the fluid bulk temperature remaining constant. Under such conditions, the heat transfer coefficients are the same as indicated by equations (6) and (8) with $\gamma = +1$.

Fig. 3 shows the coefficients plotted in the usual way for a fluid with $Pr = 0{\cdot}7$.

The relative magnitudes are readily obtainable from the curves for a given degree of asymmetry of heat transfer.

For $\gamma = -1$, equations (6) and (8) are identical, and there is good agreement between them and the empirical equation

$$Nu = 0{\cdot}023\, Re^{0{\cdot}8}\, Pr^{0{\cdot}4},$$

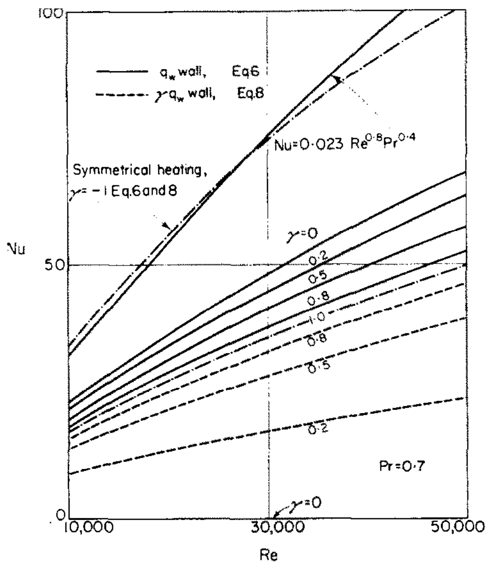

FIG. 3. Heat transfer coefficients.

which is obtained by dimensional analysis and is assumed to apply to channel flow through the hydraulic radius concept.

The use of this empirical equation as it stands, is not warranted for cases of asymmetrical heat transfer, since it does not allow for the additional parameter γ. (A modified form would be $Nu = f(\gamma, Re, Pr)$ where the function f is determined experimentally.) However, comparison of the results of the present analysis for the case of symmetrical heating (i.e. $\gamma = 1$) with the equation $Nu = 0{\cdot}023\ Re^{0{\cdot}8}\ Pr^{0{\cdot}4}$ is valid, and shows that the assumptions made regarding eddy diffusivity, etc., are warranted even over a considerable range of Re, and therefore in the general asymmetrical heating case these assumptions should be acceptable also.

The theory is valid for fluids having Pr greater than about 0·7. For very small values of Pr, the thermal diffusivity α becomes significant in the total conductivity term $(\epsilon + \alpha)$. This limits the use of the present results, which are not valid in cases of heat removal applications where the coolant fluid might have a small Pr number (e.g. liquid metals).

An experimental programme has been devised to test the predictions presented in this paper.

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